

Piezoelectricity

In 1880, Jacques and Pierre Curie discovered an unusual characteristic of certain crystalline minerals: when subjected to a mechanical force, the crystals became electrically polarized. Tension and compression generated voltages of opposite polarity, and in proportion to the applied force. Subsequently, the converse of this relationship was confirmed: if one of these voltage-generating crystals was exposed to an electric field it lengthened or shortened according to the polarity of the field, and in proportion to the strength of the field. These behaviors were labeled the piezoelectric effect and the inverse piezoelectric effect, respectively, from the Greek word *piezein*, meaning to press or squeeze.

Although the magnitudes of piezoelectric voltages, movements, or forces are small, and often require amplification (a typical disc of piezoelectric ceramic will increase or decrease in thickness by only a small fraction of a millimeter, for example) piezoelectric materials have been adapted to an impressive range of applications. The piezoelectric effect is used in sensing applications, such as in force or displacement sensors. The inverse piezoelectric effect is used in actuation applications, such as in motors and devices that precisely control positioning, and in generating sonic and ultrasonic signals.

In the 20th century metal oxide-based piezoelectric ceramics and other man-made materials enabled designers to employ the piezoelectric effect and the inverse piezoelectric effect in many new applications. These materials generally are physically strong and chemically inert, and they are relatively inexpensive to manufacture. The composition, shape, and dimensions of a piezoelectric ceramic element can be tailored to meet the requirements of a specific purpose. Ceramics manufactured from formulations of lead zirconate / lead titanate exhibit greater sensitivity and higher operating temperatures, relative to ceramics of other compositions, and "PZT" materials currently are the most widely used piezoelectric ceramics.

How are piezoelectric ceramics made?

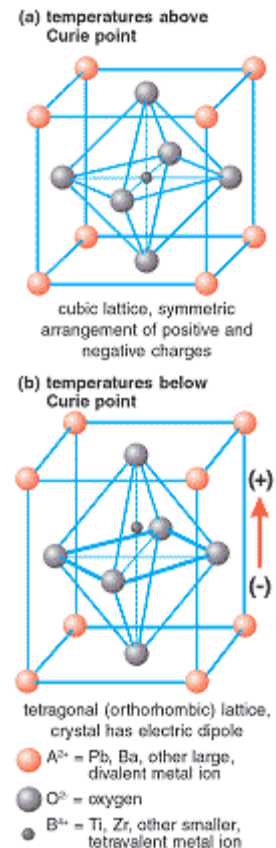
A traditional piezoelectric ceramic is a mass of perovskite crystals, each consisting of a small, tetravalent metal ion, usually titanium or zirconium, in a lattice of larger, divalent metal ions, usually lead or barium, and O^{2-} ions (Figure 1.1). Under conditions that confer tetragonal or rhombohedral symmetry on the crystals, each crystal has a dipole moment (Figure 1.1b).

To prepare a piezoelectric ceramic, fine powders of the component metal oxides are mixed in specific proportions, then heated to form a uniform powder. The powder is mixed with an organic binder and is formed into structural elements having the desired shape (discs, rods, plates, etc.). The elements are fired according to a specific time and temperature program, during which the powder particles sinter and the material attains a dense crystalline structure. The elements are cooled, then shaped or trimmed to specifications, and electrodes are applied to the appropriate surfaces.

Above a critical temperature, the *Curie point*, each perovskite crystal in the fired ceramic element exhibits a simple cubic symmetry with no dipole moment (Figure 1.1a). At temperatures below the Curie point, however, each crystal has tetragonal or rhombohedral symmetry and a dipole moment (Figure 1.1b). Adjoining dipoles form regions of local alignment called *domains*. The alignment gives a net dipole moment to the domain, and thus a net polarization. The direction of polarization among neighboring domains is random, however, so the ceramic element has no overall polarization (Figure 1.2a).

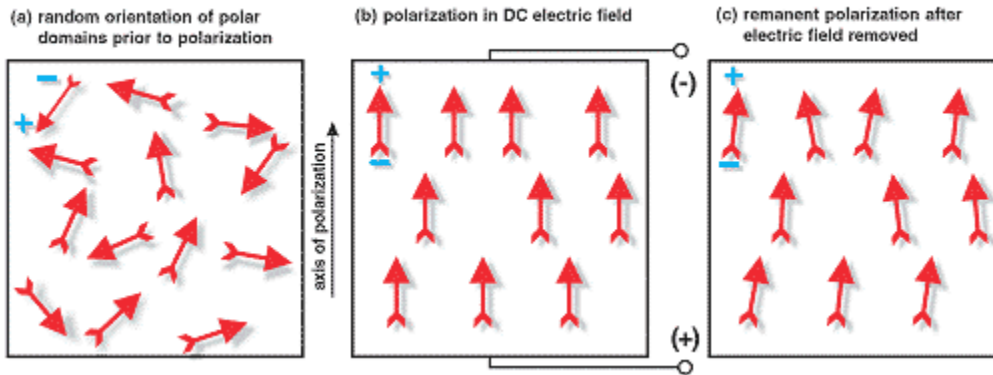
The domains in a ceramic element are aligned by exposing the element to a strong, direct current electric

Figure 1.1 Crystal structure of a traditional piezoelectric ceramic



field, usually at a temperature slightly below the Curie point (*Figure 1.2b*). Through this polarizing (*poling*) treatment, domains most nearly aligned with the electric field expand at the expense of domains that are not aligned with the field, and the element lengthens in the direction of the field. When the electric field is removed most of the dipoles are locked into a configuration of near alignment (*Figure 1.2c*). The element now has a permanent polarization, the remanent polarization, and is permanently elongated.

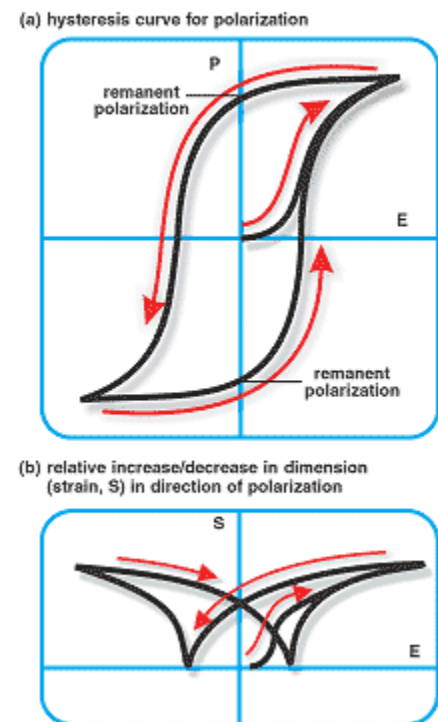
Figure 1.2 Polarizing (poling) a piezoelectric ceramic*



Analogous to corresponding characteristics of ferromagnetic materials, a poled ferroelectric material exhibits hysteresis. *Figure 1.3* shows a typical hysteresis curve created by applying an electric field to a piezoelectric ceramic element until maximum polarization, P_s , is attained, reducing the field to zero to determine the remanent polarization, P_r , reversing the field to attain a negative maximum polarization and negative remanent polarization, and re-reversing the field to restore the positive remanent polarization. The tracing below the hysteresis curve plots the relative change in the dimension of the ceramic element along the direction of polarization, corresponding to the change in the electric field. The relative increase / decrease in the dimension parallel to the direction of the electric field is accompanied by a corresponding, but approximately 50% smaller, relative decrease / increase in the dimension perpendicular to the electric field.

Figure 1.3. Effects of Electric Field (E) on Polarization (P) and Corresponding Elongation / Contraction of a Ceramic Element

Figure 1.3 Effects of electric field (E) on polarization (P) and corresponding elongation / contraction of a ceramic element



What can piezoelectric ceramics do?

Mechanical compression or tension on a poled piezoelectric ceramic element changes the dipole moment, creating a voltage. Compression along the direction of polarization, or tension perpendicular to the direction of polarization, generates voltage of the same polarity as the poling voltage (*Figure 1.4b*). Tension along the direction of polarization, or compression perpendicular to the direction of polarization, generates a voltage with polarity opposite that of the poling voltage (*Figure 1.4c*). These actions are generator actions -- the ceramic element converts the mechanical energy of compression or tension into electrical energy. This behavior is used in fuel-igniting devices, solid state batteries, force-sensing devices, and other products. Values for compressive stress and the voltage (or field strength) generated by applying stress to a piezoelectric ceramic element are linearly proportional up to a material-specific stress. The same is true for applied voltage and generated strain.

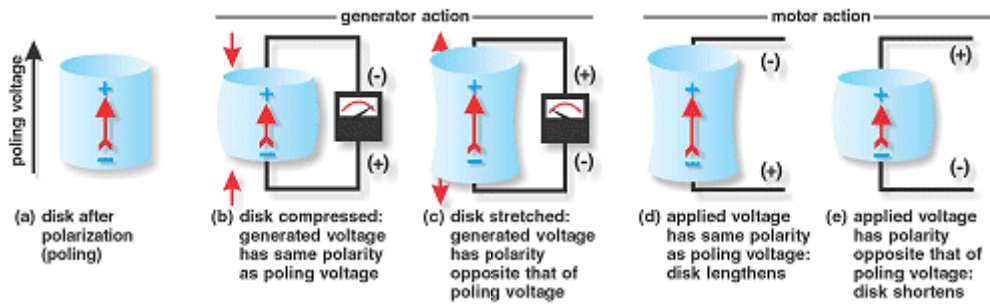
If a voltage of the same polarity as the poling voltage is applied to a ceramic element, in the direction of

the poling voltage, the element will lengthen and its diameter will become smaller (*Figure 1.4d*). If a voltage of polarity opposite that of the poling voltage is applied, the element will become shorter and broader (*Figure 1.4e*). If an alternating voltage is applied, the element will lengthen and shorten cyclically, at the frequency of the applied voltage. This is motor action -- electrical energy is converted into mechanical energy. The principle is adapted to piezoelectric motors, sound or ultrasound generating devices, and many other products.

Figure 1.4. Generator and Motor Actions of a Piezoelectric Element

Generator action is used in fuel-igniting devices, solid state batteries, and other products; motor action is adapted to piezoelectric motors, sound or ultrasound generating devices, and many other products.

Figure 1.4 Generator and motor actions of a piezoelectric element



Piezoelectric Constants

Because a piezoelectric ceramic is anisotropic, physical constants relate to both the direction of the applied mechanical or electric force and the directions perpendicular to the applied force. Consequently, each constant generally has two subscripts that indicate the directions of the two related quantities, such as stress (force on the ceramic element / surface area of the element) and strain (change in length of element / original length of element) for elasticity. The direction of positive polarization usually is made to coincide with the Z-axis of a rectangular system of X, Y, and Z axes (Figure 1.6). Direction X, Y, or Z is represented by the subscript 1, 2, or 3, respectively, and shear about one of these axes is represented by the subscript 4, 5, or 6, respectively. Definitions of the most frequently used constants, and equations for determining and interrelating these constants, are summarized here. The *piezoelectric charge constant*, d , the *piezoelectric voltage constant*, g , and the *permittivity*, ϵ , are temperature dependent factors.

Figure 1.6 Directions of forces affecting a piezoelectric element

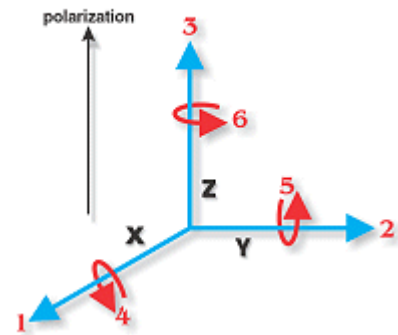


Figure 1.6 - The direction of positive polarization usually is made to coincide with the Z-axis.

Piezoelectric Charge Constant

The *piezoelectric charge constant*, d , is the polarization generated per unit of mechanical stress (T) applied to a piezoelectric material or, alternatively, is the mechanical strain (S) experienced by a piezoelectric material per unit of electric field applied. The first subscript to d indicates the direction of polarization generated in the material when the electric field, E , is zero or, alternatively, is the direction of the applied field strength. The second subscript is the direction of the applied stress or the induced strain, respectively. Because the strain induced in a piezoelectric material by an applied electric field is the product of the value for the electric field and the value for d , d is an important indicator of a material's suitability for strain-dependent (actuator) applications.

- d_{33}** induced polarization in direction 3 (parallel to direction in which ceramic element is polarized) per unit stress applied in direction 3
or
induced strain in direction 3 per unit electric field applied in direction 3
- d_{31}** induced polarization in direction 3 (parallel to direction in which ceramic element is polarized) per unit stress applied in direction 1 (perpendicular to direction in which ceramic element is polarized)
or
induced strain in direction 1 per unit electric field applied in direction 3
- d_{15}** induced polarization in direction 1 (perpendicular to direction in which ceramic element is polarized) per unit shear stress applied about direction 2 (direction 2 perpendicular to direction in which ceramic element is polarized)
or
induced shear strain about direction 2 per unit electric field applied in direction 1

Piezoelectric Voltage Constant

The *piezoelectric voltage constant*, g , is the electric field generated by a piezoelectric material per unit of mechanical stress applied or, alternatively, is the mechanical strain experienced by a piezoelectric material per unit of electric displacement applied. The first subscript to g indicates the direction of the electric field generated in the material, or the direction of the applied electric displacement. The second subscript is the direction of the applied stress or the induced strain, respectively. Because the strength of

the induced electric field produced by a piezoelectric material in response to an applied physical stress is the product of the value for the applied stress and the value for g , g is important for assessing a material's suitability for sensing (sensor) applications.

- g_{33} induced electric field in direction 3 (parallel to direction in which ceramic element is polarized) per unit stress applied in direction 3
or
induced strain in direction 3 per unit electric displacement applied in direction 3
- g_{31} induced electric field in direction 3 (parallel to direction in which ceramic element is polarized) per unit stress applied in direction 1 (perpendicular to direction in which ceramic element is polarized)
or
induced strain in direction 1 per unit electric displacement applied in direction 3
- g_{15} induced electric field in direction 1 (perpendicular to direction in which ceramic element is polarized) per unit shear stress applied about direction 2 (direction 2 perpendicular to direction in which ceramic element is polarized)
or
induced shear strain about direction 2 per unit electric displacement applied in direction 1

Permittivity

The *permittivity*, or *dielectric constant*, ϵ , for a piezoelectric ceramic material is the dielectric displacement per unit electric field. ϵ^T is the permittivity at constant stress, ϵ^S is the permittivity at constant strain. The first subscript to ϵ indicates the direction of the dielectric displacement; the second is the direction of the electric field.

The relative dielectric constant, K , is the ratio of ϵ , the amount of charge that an element constructed from the ceramic material can store, relative to the absolute dielectric constant, ϵ_0 , the charge that can be stored by the same electrodes when separated by a vacuum, at equal voltage ($\epsilon_0 = 8.85 \times 10^{-12}$ farad / meter).

- ϵ_{11}^T permittivity for dielectric displacement and electric field in direction 1 (perpendicular to direction in which ceramic element is polarized), under constant stress
- ϵ_{33}^S permittivity for dielectric displacement and electric field in direction 3 (parallel to direction in which ceramic element is polarized), under constant strain

Elastic Compliance

Elastic compliance, s , is the strain produced in a piezoelectric material per unit of stress applied and, for the 11 and 33 directions, is the reciprocal of the modulus of elasticity (Young's modulus, Y). s^D is the compliance under a constant electric displacement; s^E is the compliance under a constant electric field. The first subscript indicates the direction of strain, the second is the direction of stress.

- s_{11}^E elastic compliance for stress in direction 1 (perpendicular to direction in which ceramic element is polarized) and accompanying strain in direction 1, under constant electric field (short circuit)
- s_{33}^D elastic compliance for stress in direction 3 (parallel to direction in which ceramic element is polarized) and accompanying strain in direction 3, under constant electric displacement (open circuit)

Young's Modulus

Young's modulus, Y , is an indicator of the stiffness (elasticity) of a ceramic material. Y is determined from the value for the stress applied to the material divided by the value for the resulting strain in the same direction.

Electromechanical Coupling Factor

The *electromechanical coupling factor*, k , is an indicator of the effectiveness with which a piezoelectric material converts electrical energy into mechanical energy, or converts mechanical energy into electrical energy. The first subscript to k denotes the direction along which the electrodes are applied; the second denotes the direction along which the mechanical energy is applied, or developed.

k values quoted in ceramic suppliers' specifications typically are theoretical maximum values. At low input frequencies, a typical piezoelectric ceramic can convert 30 - 75% of the energy delivered to it in one form into the other form, depending on the formulation of the ceramic and the directions of the forces involved.

A high k usually is desirable for efficient energy conversion, but k does not account for dielectric losses or mechanical losses, nor for recovery of unconverted energy. The accurate measure of efficiency is the ratio of converted, useable energy delivered by the piezoelectric element to the total energy taken up by the element. By this measure, piezoelectric ceramic elements in well designed systems can exhibit efficiencies that exceed 90%.

The dimensions of a ceramic element can dictate unique expressions of k . For a thin disc of piezoelectric ceramic the planar coupling factor, k_p , expresses radial coupling - the coupling between an electric field parallel to the direction in which the ceramic element is polarized (direction 3) and mechanical effects that produce radial vibrations, relative to the direction of polarization (direction 1 and direction 2). For a disc or plate of material whose surface dimensions are large relative to its thickness, the thickness coupling factor, k_t , a unique expression of k_{33} , expresses the coupling between an electric field in direction 3 and mechanical vibrations in the same direction. The resonance frequency for the thickness dimension of an element of this shape is much higher than the resonance frequency for the transverse dimensions. At the same time, strongly attenuated transverse vibrations at this higher resonance frequency, a result of the transverse contraction / expansion that accompanies the expansion / contraction in thickness, make k_t lower than k_{33} , the corresponding factor for longitudinal vibrations of a thin rod of the same material, for which a much lower longitudinal resonance frequency more closely matches the transverse resonance frequency.

k_{33} factor for electric field in direction 3 (parallel to direction in which ceramic element is polarized) and longitudinal vibrations in direction 3
(ceramic rod, length >10x diameter)

k_t factor for electric field in direction 3 and vibrations in direction 3
(thin disc, surface dimensions large relative to thickness; $k_t < k_{33}$)

k_{31} factor for electric field in direction 3 (parallel to direction in which ceramic element is polarized) and longitudinal vibrations in direction 1 (perpendicular to direction in which ceramic element is polarized)
(ceramic rod)

k_p factor for electric field in direction 3 (parallel to direction in which ceramic element is polarized) and radial vibrations in direction 1 and direction 2 (both perpendicular to direction in which ceramic element is polarized)
(thin disc)

Dielectric Dissipation Factor

The *dielectric dissipation factor* (*dielectric loss factor*), $\tan \delta$, for a ceramic material is the tangent of the dielectric loss angle. $\tan \delta$ is determined by the ratio of effective conductance to effective susceptance in a parallel circuit, measured by using an impedance bridge. Values for $\tan \delta$ typically are determined at 1 kHz.

Frequency Constant

When an unrestrained piezoelectric ceramic element is exposed to a high frequency alternating electric field, an impedance minimum, the planar or radial resonance frequency, coincides with the series resonance frequency, f_s . The relationship between the *radial mode resonance frequency constant*, N_P , and the diameter of the ceramic element, D_ϕ , is expressed by:

$$N_P = f_s D_\phi$$

At higher resonance, another impedance minimum, the *axial resonance frequency*, is encountered. The thickness mode frequency constant, N_T , is related to the thickness of the ceramic element, h , by:

$$N_T = f_s h$$

A third frequency constant, the *longitudinal mode frequency constant*, is related to the length of the element:

$$N_L = f_s l$$

Most-Used Constants and Equations

Aging Rate

$$\text{Aging rate} = (\text{Par}_2 - \text{Par}_1) / ((\text{Par}_1) (\log t_2 - \log t_1))$$

Bandwidth

$$B \cong k f_p \text{ or } B \cong k f_s$$

Dielectric Constant (Relative)

permittivity of ceramic material / permittivity of free space*

$$K^T = \epsilon^T / \epsilon_0$$

*8.85 x 10⁻¹² farad / meter

Dielectric Dissipation Factor (Dielectric Loss Factor)

conductance / susceptance for parallel circuit equivalent to ceramic element;

tangent of loss angle ($\tan d$)

measure directly, typically at 1 kHz

Elastic Compliance

strain developed / stress applied;

inverse of Young' s modulus (elasticity)

$$s = 1 / \nu^2$$

$$s_{33}^D = 1 / Y_{33}^D$$

$$s_{33}^E = 1 / Y_{33}^E$$

$$s_{11}^D = 1 / Y_{11}^D$$

$$s_{11}^E = 1 / Y_{11}^E$$

Electromechanical Coupling Factor

mechanical energy converted / electric energy input

or

electric energy converted / mechanical energy input

Static / low frequencies

ceramic plate

$$k_{31}^2 = d_{31}^2 / (s_{11}^E \epsilon_{33}^T)$$

ceramic disc

$$k_p^2 = 2d_{31}^2 / ((s_{11}^E + s_{12}^E) \epsilon_{33}^T)$$

ceramic rod

$$k_{33}^2 = d_{33}^2 / (s_{33}^E \epsilon_{33}^T)$$

Higher frequencies

ceramic plate

$$k_{31}^2 = \frac{(\pi/2) (f_n / f_m) \tan [(\pi/2) ((f_n - f_m) / f_m)]}{1 + (\pi/2) (f_n / f_m) \tan [(\pi/2) ((f_n - f_m) / f_m)]}$$

ceramic disc

$$k_p \cong \sqrt{[2.51 (f_n - f_m) / f_n] - [(f_n - f_m) / f_n]^2}$$

ceramic rod

$$k_{33}^2 = (\pi/2) (f_n / f_m) \tan [(\pi/2) ((f_n - f_m) / f_n)]$$

any shape

$$k_{\text{eff}}^2 = (f_n^2 - f_m^2) / f_n^2$$

Frequency Constant

resonance frequency of linear dimension governing resonance

$$N_L \text{ (longitudinal mode)} = f_s l$$

$$N_P \text{ (radial mode)} = f_s D_\phi$$

$$N_T \text{ (thickness mode)} = f_s h$$

Mechanical Quality Factor

reactance / resistance for series circuit equivalent to ceramic element

$$Q_m = f_n^2 / (2\pi f_m C_0 Z_m (f_n^2 - f_m^2))$$

Piezoelectric Charge Constant

electric field generated by unit area of ceramic / stress applied

or

strain in ceramic element / unit electric field applied

$$d = k \sqrt{(s_{33}^E \epsilon_{33}^T)}$$

$$d_{31} = k_{31} \sqrt{(s_{11}^E \epsilon_{33}^T)}$$

$$d_{33} = k_{33} \sqrt{(s_{33}^E \epsilon_{33}^T)}$$

$$d_{15} = k_{15} \sqrt{(s_{55}^E \epsilon_{11}^T)}$$

Piezoelectric Voltage Constant

electric field generated / stress applied

or

strain in ceramic element / electric displacement applied

$$g = d / \epsilon^T$$

$$g_{31} = d_{31} / \epsilon_{33}^T$$

$$g_{33} = d_{33} / \epsilon_{33}^T$$

$$g_{15} = d_{15} / \epsilon_{11}^T$$

Young's Modulus

stress applied / strain developed

$$Y = (F / A) / (\Delta l / l) = T / S$$

Relationship among d , ϵ^T , and g

$$g = d / \epsilon^T \text{ or } d = g \epsilon^T$$

Symbols

A surface area of ceramic element (m^2)

B	bandwidth (frequency)
d	piezoelectric charge constant (C / N)
D_ϕ	diameter of ceramic disc or rod (m)
ϵ_0	permittivity of free space (8.85×10^{-12} farad / m)
ϵ^T	permittivity of ceramic material (farad / m) (at constant stress)
F	force
f_m	minimum impedance frequency (resonance frequency) (Hz)
f_n	maximum impedance frequency (anti-resonance frequency) (Hz)
f_p	parallel resonance frequency (Hz)
f_s	series resonance frequency (Hz)
g	piezoelectric voltage constant (Vm / N)
h	height (thickness) of ceramic element (m)
k	electromechanical coupling factor
k_{eff}	effective coupling factor
K^T	relative dielectric constant (at constant stress)
l	initial length of ceramic element (m)
N	frequency constant (Hz* \cdot m)
Par_1	value for parameter Par at t_1 (days)
Par_2	value for parameter Par at t_2 (days)
Q_m	mechanical quality factor
ρ	density of ceramic (kg / m^3)
s	elastic compliance (m^2 / N)
S	strain
t_1	time 1 after polarization (days)
t_2	time 2 after polarization (days)
$\tan \delta$	dielectric dissipation factor
T	stress
T°	temperature
T_C	Curie point ($^\circ\text{C}$)
v	velocity of sound in the ceramic material (m / s)
w	width of ceramic element (m)
Y	Young' s modulus (N / m^2)
Z_m	minimum impedance at f_m (ohm)

Soft Ceramics Versus Hard Ceramics

Small amounts of a *donor dopant* added to a ceramic formulation create metal (cation) vacancies in the crystal structure, enhancing the effects of extrinsic factors on the piezoelectric properties of the ceramic. The products of such formulations - *soft ceramics* - are characterized by large electromechanical coupling factors, large piezoelectric constants, high permittivity, large dielectric constants, high dielectric losses, low mechanical quality factors, and poor linearity. Soft ceramics produce larger displacements and wider signal band widths, relative to hard ceramics, but they exhibit greater hysteresis, and are more susceptible to depolarization or other deterioration. Lower Curie points (generally below 300°C) dictate that soft ceramics be used at lower temperatures. Generally large values for permittivity and dielectric dissipation factor restrict or eliminate soft ceramics from applications requiring combinations of high frequency inputs and high electric fields. Consequently, soft ceramics are used primarily in sensing applications, rather than in power applications.

An *acceptor dopant* in a ceramic formulation creates oxygen (anion) vacancies in the crystal structure. *Hard ceramics* have characteristics generally opposite those of soft ceramics, including Curie points above 300°C, small piezoelectric charge constants, large electromechanical coupling factors, and large mechanical quality factors. They also are more difficult to polarize or depolarize. Although hard ceramics generally are more stable than soft ceramics, they cannot produce the same large displacements. Hard ceramics are compatible with high mechanical loads and high voltages.

Be aware, however, that a soft ceramic can be prepared to exhibit some characteristics approaching those of a hard ceramic, or vice versa. Thus, when choosing a ceramic for a particular application, it can be useful to look beyond general categorization, and carefully compare specific characteristics.

Characteristics of Soft Ceramics and Hard Ceramics Compared

Characteristic	Soft Ceramic	Hard Ceramic
Piezoelectric Constants	larger	smaller
Permittivity	higher	lower
Dielectric Constants	larger	smaller
Dielectric Losses	higher	lower
Electromechanical Coupling Factors	larger	smaller
Electrical Resistance	very high	lower
Mechanical Quality Factors	low	high
Coercive Field	low	higher
Linearity	poor	better
Polarization / Depolarization	easier	more difficult

Behavior of a Piezoelectric Ceramic Element

Influence of Input Frequency

At low input frequencies, the relationships between a force applied to a piezoelectric ceramic element and the electric field or charge produced by the element are:

$$E = -(g_{33}T)$$

$$Q = -(d_{33}F)$$

where E: electric field

g_{33} : piezoelectric voltage constant

T: stress on ceramic element

Q: generated charge

d_{33} : piezoelectric charge constant

F: applied force

The relationships between an applied voltage or electric field and the corresponding increase or decrease in a ceramic element's thickness, length, or width are:

$$\Delta h = d_{33}V$$

$$S = d_{33}E$$

$$\Delta l / l = d_{31}E$$

$$\Delta w / w = d_{31}E$$

where l: initial length of ceramic element

w: initial width of ceramic element

Δh : change in height (thickness) of ceramic element

Δl : change in length of ceramic element

Δw : change in width of ceramic element

d: piezoelectric charge constant

V: applied voltage

S: strain (change in height / original height of element)

E: electric field

A piezoelectric ceramic element exposed to an alternating electric field changes dimensions cyclically, at the frequency of the field. The frequency at which the element vibrates most readily in response to the electrical input, and most efficiently converts the electrical energy input into mechanical energy -- the *resonance frequency* -- is determined by the composition of the ceramic material and by the shape and volume of the element.

As the frequency of cycling is increased, the element's oscillations first approach a frequency at which impedance is minimum (maximum admittance). This frequency also is the resonance frequency. As the frequency is further increased, impedance increases to a maximum (minimum admittance), which also is the *anti-resonance frequency*. These frequencies are determined by experiment - to see how, refer to [Determining Resonance Frequency](#).

The values for minimum impedance frequency and maximum impedance frequency can be used to calculate the *electromechanical coupling factor*, k, an indicator of the effectiveness with which a piezoelectric material converts electrical energy into mechanical energy or mechanical energy into electrical energy. k depends on the mode of vibration and the shape of the ceramic element. Dielectric losses and mechanical losses also affect the efficiency of energy conversion. Dielectric losses usually are more significant than mechanical losses.

Stability - Most properties of a piezoelectric ceramic element erode gradually, in a logarithmic relationship with time after polarization. Exact rates of aging depend on the composition of the ceramic element and the manufacturing process used to prepare it. Mishandling the element by exceeding its electrical, mechanical, or thermal limitations can accelerate this inherent process.

Electrical Limitations - Exposure to a strong electric field, of polarity opposite that of the polarizing field, will depolarize a piezoelectric material. The degree of depolarization depends on the grade of material, the exposure time, the temperature, and other factors, but fields of 200-500 V / mm or greater typically have a significant depolarizing effect. An alternating current will have a depolarizing effect during each half cycle in which polarity is opposite that of the polarizing field.

Mechanical Limitations Mechanical stress sufficient to disturb the orientation of the domains in a piezoelectric material can destroy the alignment of the dipoles. Like susceptibility to electrical depolarization, the ability to withstand mechanical stress differs among the various grades and brands of piezoelectric materials.

Thermal Limitations - If a piezoelectric ceramic material is heated to its Curie point, the domains will become disordered and the material will be depolarized. The recommended upper operating temperature for a ceramic usually is approximately half-way between 0°C and the Curie point. Within the recommended operating temperature range, temperature-associated changes in the orientation of the domains are reversible. On the other hand, these changes can create charge displacements and electric fields. Also, sudden temperature fluctuations can generate relatively high voltages, capable of depolarizing the ceramic element. A capacitor can be incorporated into the system to accept the superfluous electrical energy.

For a particular ceramic material, the *pyroelectric charge constant* - the change in polarity for a given change in temperature - and the *pyroelectric field strength constant* - the change in electric field for a given change in temperature - are indicators of the vulnerability of the material to pyroelectric effects. A high piezoelectric charge constant : pyroelectric charge constant ratio or piezoelectric voltage constant : pyroelectric field strength constant ratio indicates good resistance to pyroelectric effects.

Determining Resonance Frequency

When exposed to an AC electric field, a piezoelectric ceramic element changes dimensions cyclically, at the cycling frequency of the field. The frequency at which the ceramic element vibrates most readily, and most efficiently converts the electrical energy input into mechanical energy, is the *resonance frequency*.

The pattern of an element's responses is depicted in *Figure 1.8*. As the frequency of cycling is increased, the element's oscillations first approach a frequency at which impedance is minimum (maximum admittance). This *minimum impedance frequency*, f_m , approximates the *series resonance frequency*, f_s , the frequency at which impedance in an electrical circuit describing the element is zero, if resistance caused by mechanical losses is ignored. The minimum impedance frequency also is the resonance frequency, f_r . The composition of the ceramic material and the shape and volume of the element determine the resonance frequency -- generally, a thicker element has a lower resonance frequency than a thinner element of the same shape.

As the cycling frequency is further increased, impedance increases to a maximum (minimum admittance). The maximum impedance frequency, f_n , approximates the *parallel resonance frequency*, f_p , the frequency at which parallel resistance in the equivalent electrical circuit is infinite if resistance caused by mechanical losses is ignored. The maximum impedance frequency also is the *anti-resonance frequency*, f_a . Maximum response from the element will be at a point between f_m and f_n .

Values for minimum impedance frequency, f_m , and maximum impedance frequency, f_n , can be determined by measurement. *Figure 1.8* shows a system designed to ascertain these values, and summarizes the procedure.

Figure 1.8. Impedance as a Function of Cycling Frequency

A ceramic element's oscillations first approach the minimum impedance frequency (f_m) / resonance frequency (f_r), at which the element vibrates most readily, and most efficiently converts electrical energy into mechanical energy. As cycling frequency is further increased, impedance increases to the maximum impedance frequency (f_n) / anti-resonance frequency (f_a).

Figure 1.8 Impedance as a function of frequency

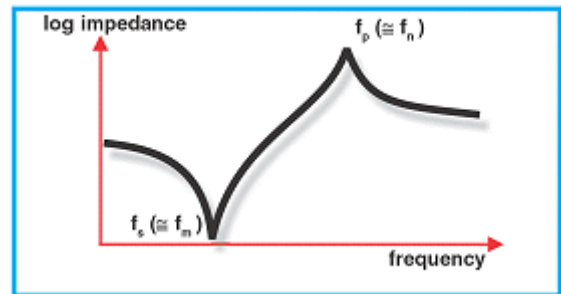
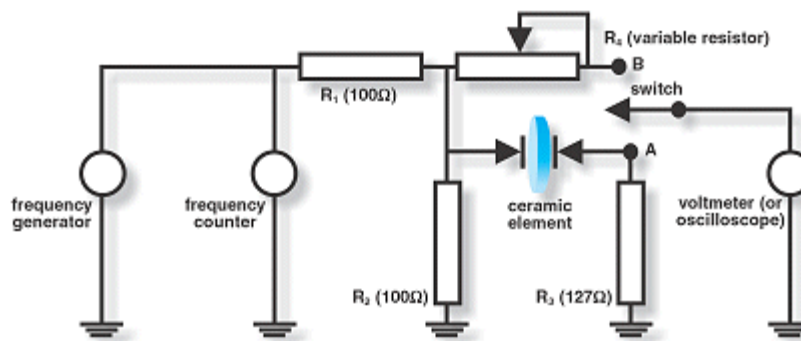


Figure 1.10. System for Determining Minimum Impedance (Resonance Frequency) and Maximum Impedance (Anti-Resonance Frequency) of a Piezoelectric Ceramic Element



Procedure:

1. Set the switch to A.
2. Place the ceramic element into position.
3. Adjust the frequency generator to give a maximum voltage value on the voltmeter. This value is the resonance frequency.
4. Set the switch to B.
5. Adjust R4 to give a voltage value on the voltmeter equal to the value in step 3. This value is the impedance resonance (Z_r).
6. Set the switch to A.
7. Adjust the frequency generator to give a minimum voltage value on the voltmeter. This value is the antiresonance frequency.

f_m and f_n can be used to calculate the electromechanical coupling factor, k . k depends on the mode of vibration and the shape of the ceramic element. The relationships between k and f_m and f_n for a ceramic plate, a disc (surface dimensions large, relative to thickness), or a rod are:

Coupling Factor for Plates / k_{31} *

$$k_{31}^2 = \frac{(\pi / 2) (f_n / f_m) \tan [(\pi / 2) ((f_n - f_m) / f_m)]}{1 + (\pi / 2) (f_n / f_m) \tan [(\pi / 2) ((f_n - f_m) / f_m)]}$$

* electric field parallel to direction of polarization,
induced strain perpendicular to direction of polarization

Coupling Factor for Discs / k_p **

$$k_p \cong \sqrt{[2.51 (f_n - f_m) / f_n] - ((f_n - f_m) / f_n)^2}$$

** electric field parallel to direction of polarization,
induced strain in same direction

Coupling Factor for Rods / k_{33} ***

$$k_{33}^2 = (\pi / 2) (f_n / f_m) \tan [(\pi / 2) ((f_n - f_m) / f_n)]$$

*** electric field parallel to direction of polarization,
induced strain in same direction

Applications

A piezoelectric system can be constructed for virtually any application for which any other type of electromechanical transducer can be used. For any particular application, however, limiting factors include the size, weight, and cost of the system. Piezoceramic devices fit into four general categories: *generators*, *sensors*, *actuators*, and *transducers*. Characteristics of each group are briefly summarized here.

Generators

Piezoelectric ceramics can generate voltages sufficient to spark across an electrode gap, and thus can be used as ignitors in fuel lighters, gas stoves, welding equipment, and other such apparatus. Piezoelectric ignition systems are small and simple -- distinct advantages relative to alternative systems that include permanent magnets or high voltage transformers and capacitors.

Alternatively, the electrical energy generated by a piezoelectric element can be stored. Techniques used to make multilayer capacitors have been used to construct multilayer piezoelectric generators. Such generators are excellent solid state batteries for electronic circuits.

Sensors

A sensor converts a physical parameter, such as acceleration or pressure, into an electrical signal. In some sensors the physical parameter acts directly on the piezoelectric element; in other devices an acoustical signal establishes vibrations in the element and the vibrations are, in turn, converted into an electrical signal. Often, the system provides a visual, audible, or physical response to the input from the sensor -- automobile seatbelts lock in response to a rapid deceleration, for example.

Actuators

A piezoelectric actuator converts an electrical signal into a precisely controlled physical displacement, to finely adjust precision machining tools, lenses, or mirrors. Piezoelectric actuators also are used to control hydraulic valves, act as small-volume pumps or special-purpose motors, and in other applications. Piezoelectric motors are unaffected by energy efficiency losses that limit the miniaturization of electromagnetic motors, and have been constructed to sizes of less than 1 cm³. A potentially important additional advantage to piezoelectric motors is the absence of electromagnetic noise.

Alternatively, if physical displacement is prevented, an actuator will develop a useable force.

Transducers

Piezoelectric transducers convert electrical energy into vibrational mechanical energy, often sound or ultrasound, that is used to perform a task.

Piezoelectric transducers that generate audible sounds afford significant advantages, relative to alternative electromagnetic devices -- they are compact, simple, and highly reliable, and minimal energy can produce a high level of sound. These characteristics are ideally matched to the needs of battery-powered equipment.

Because the piezoelectric effect is reversible, a transducer can both generate an ultrasound signal from electrical energy and convert incoming sound into an electrical signal. Some devices designed for measuring distances, flow rates, or fluid levels incorporate a single piezoelectric transducer in the signal sending and receiving roles, other designs incorporate two transducers and separate these roles.

Piezoelectric transducers also are used to generate ultrasonic vibrations for cleaning, atomizing liquids, drilling or milling ceramics or other difficult materials, welding plastics, medical diagnostics, or for other purposes.

New Materials

In most applications for piezoelectric materials there is continual demand for improved performance: greater movement, higher temperature limits, longer lifetime, etc. *Single crystals* and *relaxor materials* are among the subjects of current research.

Single Crystals

Single crystals of natural or man-made materials exhibit the desirable piezoelectric properties that might be offered by a polycrystalline ceramic element if all of its domains were perfectly aligned. An expanding variety of single crystals is being developed for acoustical, optical, wireless communication, and other applications. Materials used to fabricate single-crystal piezoelectric elements include lead magnesium niobate / lead titanate (PMN-PT), lead zirconate niobate / lead titanate (PZN-PT), lithium niobate (LiNbO_3), lithium niobate with dopants, lithium tetraborate ($\text{Li}_2\text{B}_4\text{O}_7$), and quartz. Barium titanate (BaTiO_3) is a potential non-lead source of piezoelectric crystals for low temperature and room temperature applications. Single-crystal PMN-PT and PZN-PT elements exhibit ten times the strain of comparable polycrystalline lead-zirconate-titanate elements.





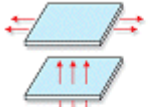

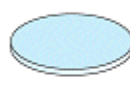




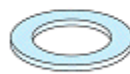


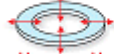
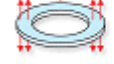
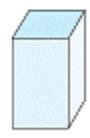



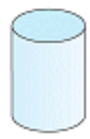



Applications for single-crystal materials include actuators and diagnostic and therapeutic medical devices. A useful combination of piezoelectric and electro-optic properties makes lithium niobate and doped lithium niobate crystals very useful for surface acoustic wave (SAW) devices and electro-optical applications. A SAW chip made from a lithium tetraborate crystal can be significantly smaller than its lithium niobate or quartz counterpart. Other applications for lithium tetraborate crystals include bulk acoustic wave (BAW) devices, pagers, cordless and cellular telephones, and data communication devices. Applications for quartz crystals include timing mechanisms for watches and clocks and delay lines for electrical circuits. The performance of a single-crystal element depends on the direction in which the raw crystal is cut: a cut normal to the x axis will produce maximal potential for expanding in thickness; a crystal cut normal to the y axis will have maximal potential for shear distortion.

Relaxors

In relaxor materials, the transition between piezoelectric behavior and loss of piezoelectric capability does not occur at a specific temperature (Curie point), but instead occurs over a temperature range (Curie range). In addition to relative insensitivity to temperature, single crystals of some relaxor formulations exhibit very high electromechanical coupling factors -- values greater than 0.9, versus values of 0.7-0.8 for conventional lead-zirconate-titanate ceramics. This combination makes relaxors very attractive materials for actuator, transducer, and other applications. Lead magnesium niobate, lead magnesium niobate / lanthanum formulations, and lead nickel niobate currently are among the most studied relaxor materials.


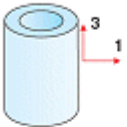


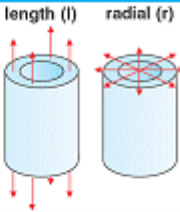
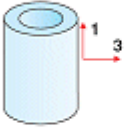
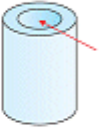
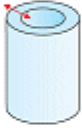
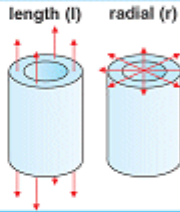
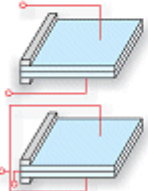


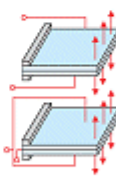
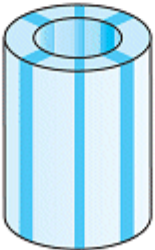
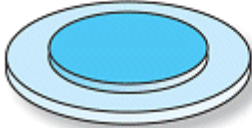
Modes of Vibration

Table 1.8 Modes of vibration for common piezoelectric ceramic shapes

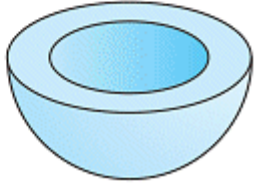
Axis	Polarization Direction	Applied Field: Voltage Output	Mode of Vibration: Displacement
			
Plate			
			<p>length or transverse (l or w)</p>  <p>thickness (h)</p> 
Disc			
			<p>radial (r)</p>  <p>thickness (h)</p> 
Ring			
			<p>radial (r)</p>  <p>thickness (h)</p> 
Bar			
			<p>length (l)</p> 
Rod			
			

Frequency Constant	Capacitance	Static Displacement	Static Voltage
$N_L = fl$ $N_L = f_r w$ $N_r = fh$	$C_s = \frac{K_{33}^T \epsilon_0 l w}{h}$	$\Delta l = \frac{d_{31} V l}{h}$ $\Delta w = \frac{d_{31} V w}{h}$ $\Delta h = d_{33} V$	$V = \frac{g_{31} F_1}{l}$ $V = \frac{g_{31} F_2}{w}$ $V = \frac{g_{33} F_3 h}{l w}$
$N_p = 2fr$ $N_r = fh$	$C_s = \frac{K_{33}^T \epsilon_0 \pi r^2}{h}$	$\Delta r = \frac{2d_{31} V r}{h}$ $\Delta h = d_{33} V$	$V = \frac{g_{31} F_r}{2\pi r}$ $V = \frac{g_{33} F_3 h}{\pi r^2}$
$N_p = f_r(OD - ID)$ $N_r = fh$	$C_s = \frac{K_{33}^T \epsilon_0 \pi (OD^2 - ID^2)}{4h}$	$\Delta r = \frac{d_{31} V (OD - ID)}{2h}$ $\Delta h = d_{33} V$	$V = \frac{g_{31} F_r}{2\pi(OD - ID)}$ $V = \frac{4g_{33} F_3 h}{\pi(OD^2 - ID^2)}$
$N_{total} = fl$	$C_s = \frac{K_{33}^T \epsilon_0 w h}{l}$	$\Delta l = d_{33} V$	$V = \frac{g_{33} F_3 l}{w h}$ $V = \frac{g_{31} F_1}{w}$ $V = \frac{g_{31} F_2}{h}$
$N_{total} = fl$	$C_s = \frac{K_{33}^T \epsilon_0 \pi r^2}{4l}$	$\Delta l = d_{33} V$	$V = \frac{g_{33} F_3 l}{\pi r^2}$ $V = \frac{g_{31} F_r}{2\pi r}$

Table 1.8 Modes of vibration for common piezoelectric ceramic shapes (cont'd)

Axis	Polarization Direction	Applied Field: Voltage Output	Mode of Vibration: Displacement
			
Ring			
			
Cylinder Wall Electrode			
			
Plate Bender			
			
<p data-bbox="170 1052 495 1077">Cylinder with Striped Electrodes</p> 	<p data-bbox="641 1052 771 1077">Disk Bender</p> 		

Frequency Constant	Capacitance	Static Displacement	Static Voltage
$N_L = f_l l$ $N_p = f_r (OD - ID)$	$C_s = \frac{K^T \epsilon_0 \pi (OD^2 - ID^2)}{4l}$	$\Delta l = d_{31} V$ $\Delta r = \frac{d_{31} V (OD - ID)}{2l}$	$V = \frac{4g_{33} F_3 l}{\pi (OD^2 - ID^2)}$ $V = \frac{g_{31} F_r}{2\pi (OD - ID)}$
$N_{max} = f_l$ $N_T = f_r (OD - ID) / 2$	$C_s = \frac{2K^T \epsilon_0 \pi l}{\ln (OD/ID)}$	$\Delta l = \frac{2d_{31} V l}{(OD - ID)}$ $\Delta r = d_{31} V$	$V = \frac{g_{31} F_1 (OD - ID)}{l(OD - ID)}$ $V = \frac{g_{33} F_3 (OD - ID)}{2(OD^2 + ID^2)}$
$N_L = \frac{3f_l^2}{h}$ $N_L = \frac{3f_r^2}{h}$	$C_s = \frac{K^T \epsilon_0 l w}{h}$	$\Delta h = \frac{3d_{31} V l^2}{2h^2}$ $\Delta h = \frac{3d_{31} V l^2}{h^2}$	$V = \frac{3g_{31} F_3 l}{2wh}$ $V = \frac{3g_{31} F_3 l}{4wh}$

Hemisphere	General
	<p>Equations valid for:</p> <p>(A) plate, disc & ring where $r, l, \& w \gg h$</p> <p>(B) bar where $l \gg r, w, \& h$ (5 to 10X)</p> <p>(C) cylinder & rod where $h \gg r$ (5 to 10X)</p> <p>(D) plate bender equations are for fully electroded cantilever mount. For beam mount multiply f_r by 2.8, Δr by 0.25, & V by 0.25</p> <p>Constants g_{31} & d_{31} are negative values resulting in negative strain & negative voltage. All variables are in metric (MKS) units. Each material has voltage, stress, and temperature limitations.</p>

Symbols

- C_s capacitance
- d piezoelectric charge constant (C / N)
- ϵ_0 permittivity of free space (8.85×10^{-12} farad / m)
- f_r resonance frequency (minimum impedance frequency) (Hz)
- F force
- g piezoelectric voltage constant (Vm / N)
- h height (thickness) of ceramic element (m)
- ID inside diameter of ceramic element (m)
- K^T relative dielectric constant (constant stress)

- I** length of ceramic element (m)
- N** frequency constant (Hz-m)
- OD** outside diameter of ceramic element (m)
- r** radius of ceramic element (m)
- V** voltage
- w** width of ceramic element (m)

Catalog No.: 90-1015

Piezoelectric Theory and Applications

The information in the Piezo Theory section of our website was abstracted from our book *Piezoelectric Ceramics: Principles and Applications*.

Subjects discussed in this applications book include:

- λ principles of piezoelectricity and behavior of piezoelectric ceramic elements
- λ fundamental mathematics of piezoelectricity
- λ traditional and experimental applications for piezoelectric materials, and related physical principles for each application: audible sound producers, flow meters, fluid level sensors, motors, pumps, delay lines, transformers, other apparatus
- λ introduction to single crystals, composites, and other latest-generation piezoelectric materials

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